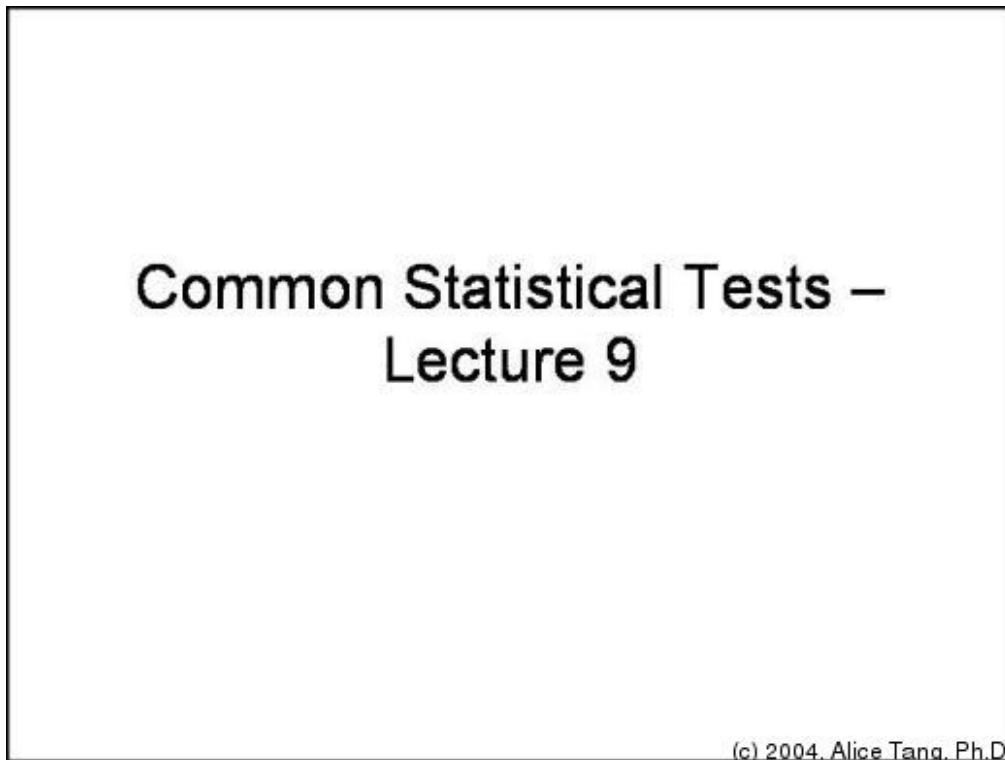
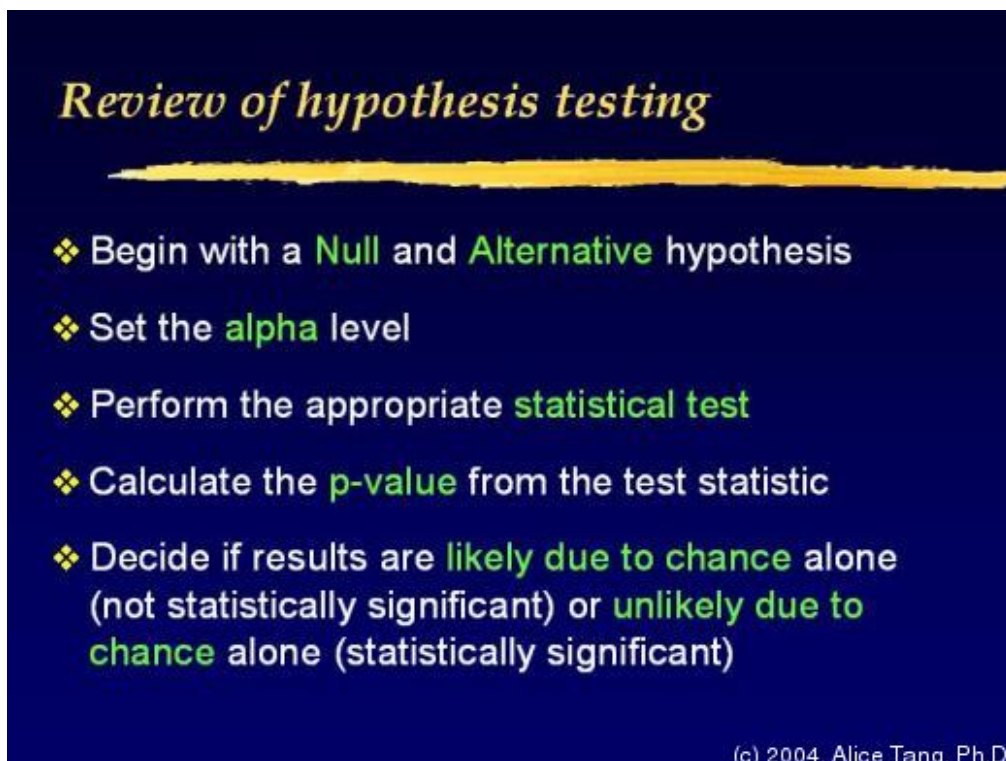


1. Lecture 9 - Introduction Slide



2. Review of hypothesis testing



3. Statistical Tests

Statistical Tests

- ❖ Statistical tests are mathematical formulas that produce p-values which allow investigators to assess the likelihood that chance accounts for the results observed in the study
- ❖ There are many different statistical tests. The choice of which test to use depends on several factors:
 - ▶ The type of data (continuous, nominal etc.)
 - ▶ The distribution of the data (normally distributed or not)
 - ▶ The type of study design (means, proportions, # of groups etc.)

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4. Statistical Tests, cont.

Statistical Tests (cont'd)

- ❖ All test statistics follow this general format:

$$\text{Test statistic} = \frac{\text{Observed value} - \text{Expected value (H}_0\text{)}}{\text{Standard Error}}$$

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5. Analysis of Continuous Data: 1-sample Z-test 1-sample t-test ...

Analysis of Continuous Data:

1-sample Z-test
1-sample t-test
2-sample t-test
Paired t-test

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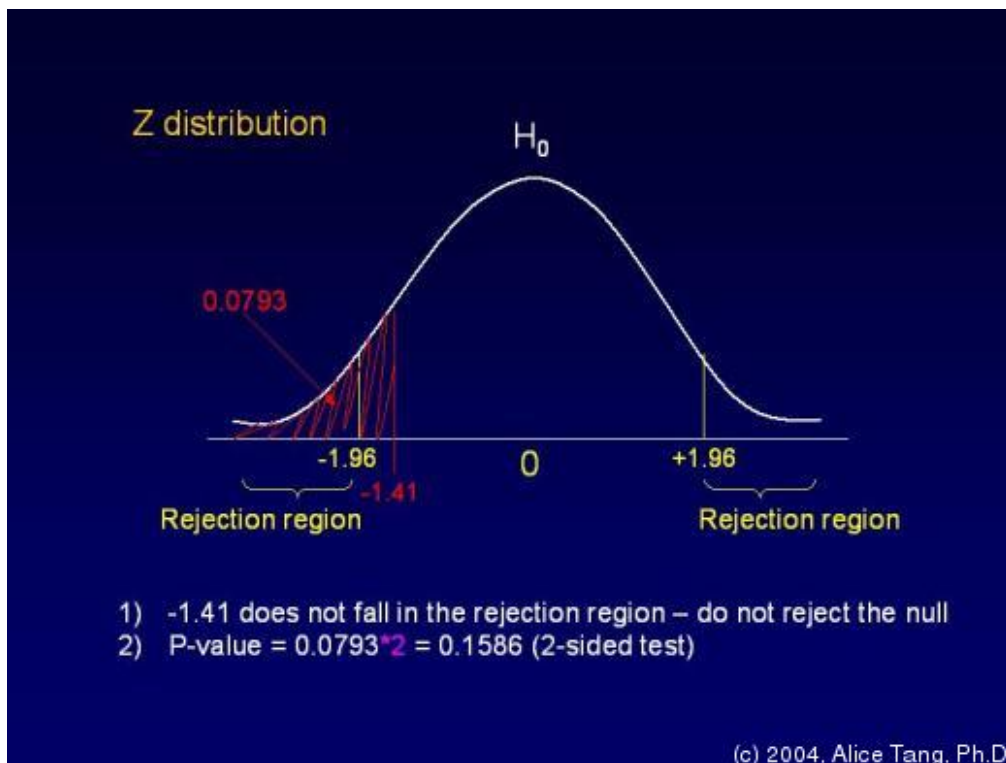
6. Example: One-sample Z-test

Example: One-sample Z-test

- ❖ Researchers are interested in whether the mean level of enzyme A in a certain population is different from 25. They measure levels of enzyme A in a sample of 10 individuals and find that the mean, $\bar{x} = 22$. Assume that the population has a known standard deviation, $\sigma = 6.7$.
- ❖ $H_0: \mu = 25$ $\alpha = 0.05$
 $H_A: \mu \neq 25$
- ❖ Calculate test statistic: $Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} = \frac{22 - 25}{6.7 / \sqrt{10}} = -1.41$
- ❖ Critical value (from Z-table): ± 1.96

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7. z distribution



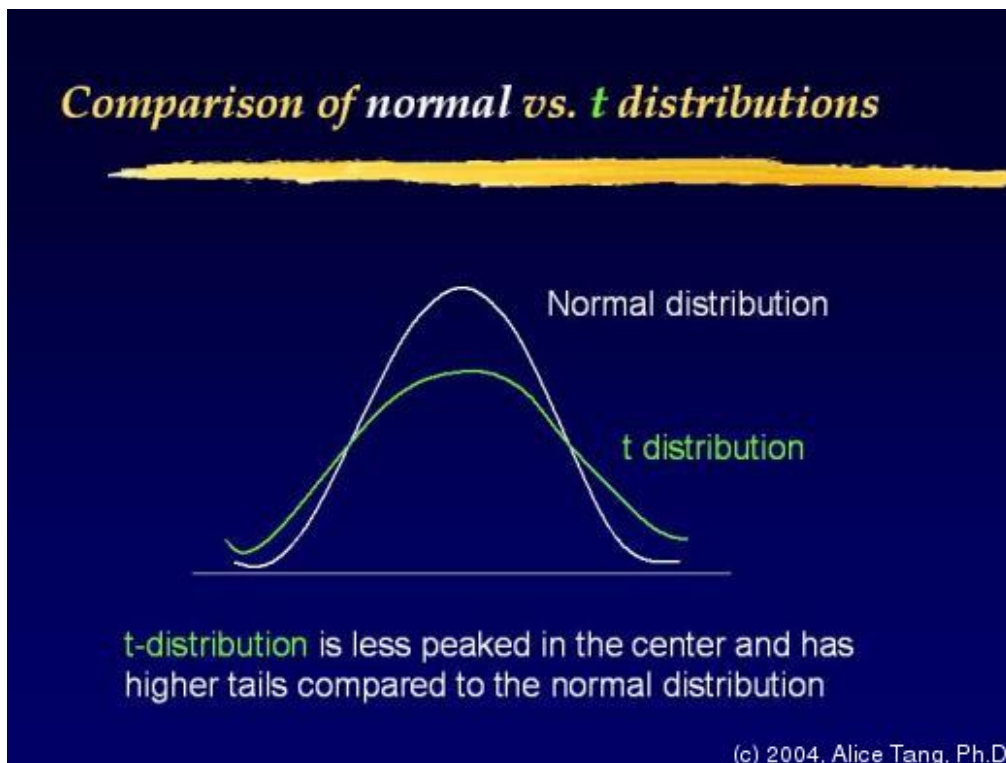
8. t-tests

t-tests

- ❖ Z-test assumes population variance (or standard deviation) is known
- ❖ When population standard deviation (σ) is unknown, but sample size is large (i.e. >30), then use sample standard deviation (s) to estimate σ and use normal distribution theory (Central Limit Theorem)
- ❖ However, when σ is unknown and sample size is small, then must use Student's t distribution theory

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9. Comparison of normal vs. t distributions



10. Properties of the t-distribution

Properties of the t-distribution

- ❖ Family of distributions – different distribution for each sample value of $n-1$ (degrees of freedom)
- ❖ It has a mean of 0 and is symmetrical around the mean
- ❖ The t-distribution approaches the normal distribution as $n-1$ approaches 30.

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11.

Example: One-sample t-test

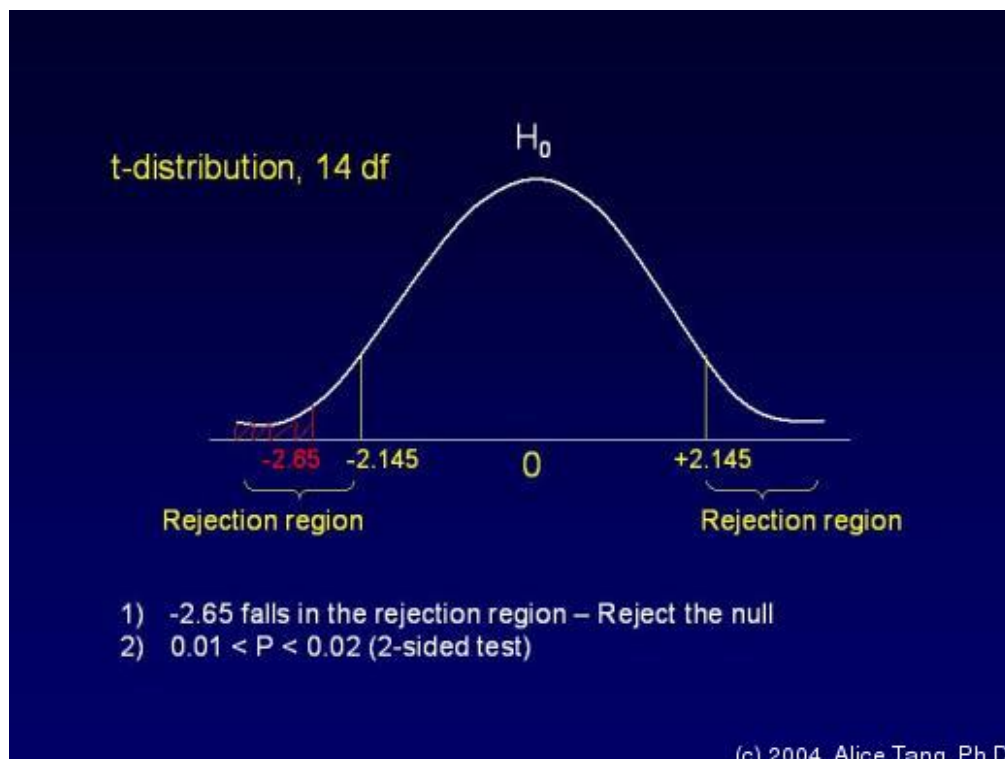
Example: One-sample t-test

- ❖ Researchers are interested in whether the mean level of enzyme B in a certain population is different from 120. They measure levels of enzyme B in a sample of 15 individuals and find that the mean, $\bar{x} = 96$ and the sample standard deviation, $s = 35$.
- ❖ $H_0: \mu = 120$ $\alpha = 0.05$
 $H_A: \mu \neq 120$
- ❖ Calculate test statistic: $t = \frac{\bar{X} - \mu_0}{s / \sqrt{n}} = \frac{96 - 120}{35 / \sqrt{15}} = -2.65$
- ❖ Critical value (t-distribution w/ 14 df) = ± 2.145

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12.

t-distribution, 14 df



13. Two-sample t-test

Two-sample t-test

- ❖ Purpose is to compare the means of a continuous variable in two independent samples
- ❖ $H_0: \mu_1 - \mu_2 = 0$
 $H_A: \mu_1 - \mu_2 \neq 0$
- ❖ Calculate test statistic:
$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\text{Sqrt} [s_p^2/n_1 + s_p^2/n_2]}$$

where,
$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$
- ❖ Critical Value = t-dist, with df = $n_1 + n_2 - 2$

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14. Example: two-sample t-test

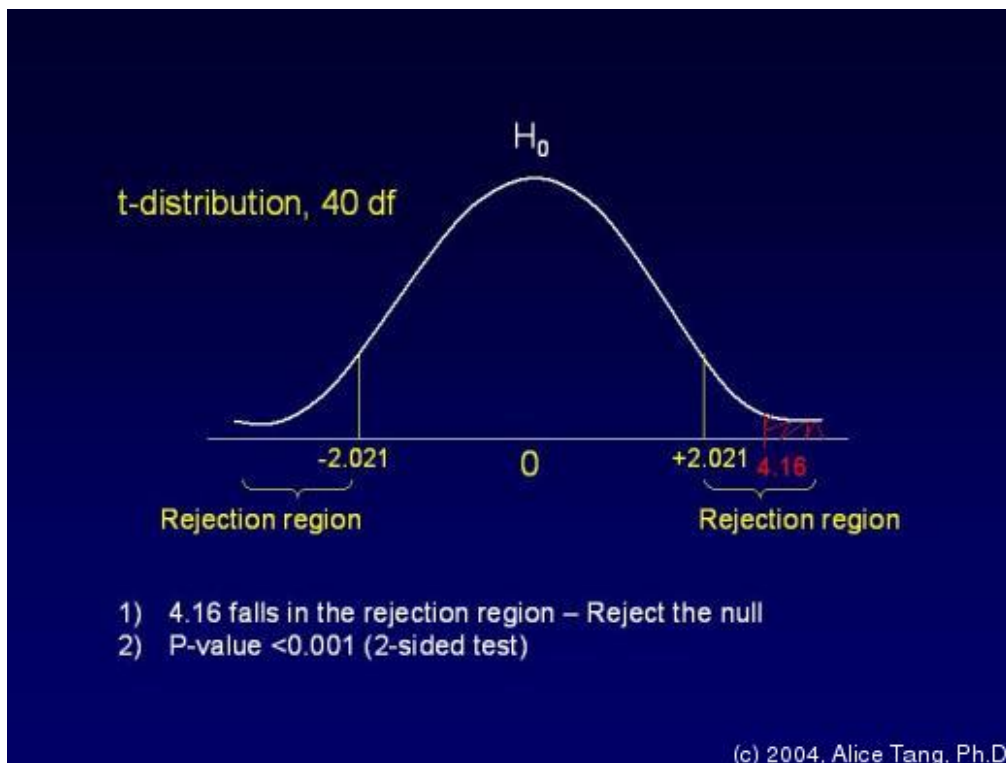
Example: two-sample t-test

- ❖ Researchers are interested in knowing whether people with diabetes have the same SBP as people without diabetes.
- ❖ $H_0: \mu_1 - \mu_2 = 0$ $\alpha = 0.05$
 $H_A: \mu_1 - \mu_2 \neq 0$
- ❖ Data:
 - ▶ Diabetics: $n_1=20, \bar{x}_1=135, s_1=10$ mm Hg
 - ▶ Nondiabetics: $n_2=25, \bar{x}_2=125, s_2=6$ mm Hg
- ❖ Test statistic: $t=(135-125)/2.4 = 4.16$
- ❖ Critical value (t-distribution w/ 43 df): ± 2.021
(use 40 df from t-table)

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15.

t-distribution, 40 df



16.

Paired t-test

Paired t-test

- ❖ **Purpose** is to compare means of two non-independent samples. For example, measurements on the same individuals before and after a treatment
- ❖ Analysis performed on **differences** between individual pairs of observations
- ❖ $H_0: \mu_d = 0$
 $H_A: \mu_d \neq 0$
- ❖ **Calculate test statistic:** $t = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}}$
- ❖ **Critical Value** = t-dist. with n-1 df

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17. Example: Paired t-test

Example: Paired t-test

- ❖ 12 subjects participated in a study on the effectiveness of a certain diet on serum cholesterol levels.
- ❖ Data:

Subject	Before	After	Difference
1	201	200	-1
2	231	236	+5
3	221	216	-5
4	260	233	-27
5	228	224	-4
etc.	etc.	etc.	etc.

$\mu_d = -20.17, s_d = 23.13$

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18. Example: paired t-test (cont'd)

Example: paired t-test (cont'd)

- ❖ Calculate test statistic:

$$t = \frac{-20.17 - 0}{23.13 / \sqrt{12}} = \frac{-20.17}{6.68} = -3.02$$

- ❖ Critical value (t-dist w/ 11 df) = ± 2.201
- ❖ -3.02 falls within the rejection region – Reject the null
- ❖ $0.01 < p < 0.02$
- ❖ Conclude that diet was effective

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19. What if your data are not normally distributed and your samp...

What if your data are not normally distributed and your sample size is relatively small?

- ❖ Mathematically transform the data into a normal distribution (take the log or square root of all values)
- ❖ Use a different class of tests called '**non-parametric**' tests. These tests are based on the ranking or ordering of data rather than their numerical values.

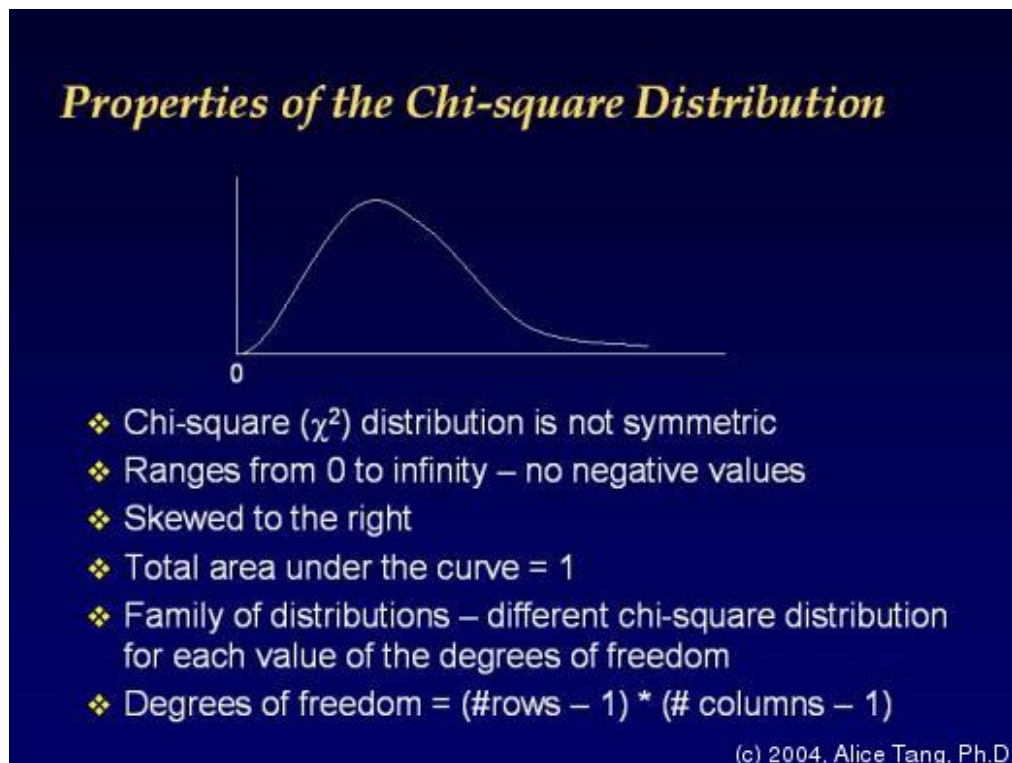
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20. Analysis of Nominal Data: Chi-square test

*Analysis of Nominal Data:
Chi-square test*

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21. Properties of the Chi-square Distribution



22. Chi-square test

Chi-square test

- ❖ Used when both exposure and outcome are categorical (nominal)... can be 2 x 2 or larger tables
- ❖ **Null hypothesis:** no difference in proportions between groups
- ❖ **Test statistic:**
$$\chi^2 = \sum \frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}} \quad (\text{in each cell})$$

where, **Expected** = (row total x column total) / TOTAL population
- ❖ **Critical Value:** χ^2 with (r-1)(c-1) degrees of freedom

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23. Example: Chi-square test

Example: Chi-square test

- ❖ In an air pollution study, a random sample of 200 households were selected from each of 2 communities. A respondent in each household was asked whether or not anyone in the household was bothered by air pollution
- ❖ Data:

Bothered by air pollution?	Community		Total
	A	B	
Yes	43	81	124
No	157	119	276
Total	200	200	400

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24. Hypotheses for Chi-square example

Hypotheses for Chi-square example

- ❖ H_0 : proportion bothered by air pollution in Community A = proportion bothered by air pollution in Community B
- ❖ H_A : proportion bothered by air pollution in Community A \neq proportion bothered by air pollution in Community B
- ❖ Alpha = 0.05

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25. Example: Chi-square test, cont. 1

Example: Chi-square test (cont'd)

Bothered by air pollution?	Community		Total
	A	B	
Yes	43	81	124
No	157	119	276
Total	200	200	400

Expected = (row total x column total) / total pop.
 $(124 \times 200) / 400 = 62$ OR $(124 / 400) \times 200 = 62$

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26. Example: Chi-square test, cont. 2

Example: Chi-square test (cont'd)

Bothered by air pollution?	Community		Total
	A	B	
Yes	43	81	124
No	157	119	276
Total	200	200	400

$$\chi^2 = \sum [(Obs - Exp)^2 / Exp]$$

$$= (43-62)^2 / 62 + (81-62)^2 / 62 + (157-138)^2 / 138 + (119-138)^2 / 138 = 16.88$$

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27.

Example: Chi-square test, cont. 3

